Problem Set 3

Problem 1.

a) Show that in a first-price auction with N bidders, whose valuations are *iid* on the [0, 1] interval, the equilibrium bid function is
$b(v_i) = v_i(N - 1)/N$
b) What happens to the auctioneer's expected revenue as N goes to infinity.
c) Show that under the above assumptions, the first-price and the second-price auctions are revenue-equivalent (recall that in the second-price auction, a weakly dominant strategy for each player is to bid her true valuation).

Problem 2. A principal-agent problem.
Caution: this is not a typical mechanism design problem, as the message sent by the agent is not observable by the principal.

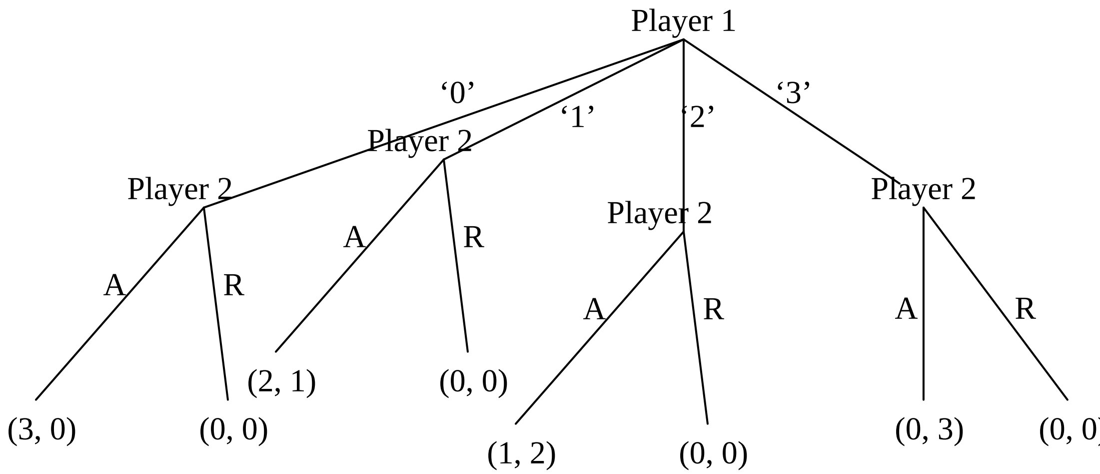
There is a principal and an agent. The principal hires an agent to do some task. The task requires some effort. The agent could choose a low level or a high level of effort. In either case, the task could bring the principal a low profit (Π_1) or a high profit (Π_2). However, a high level of effort increases the probability of the task bringing a high profit, as described in the table below ($\Pi_2 > \Pi_1$, $x > y$):

High effort			**Low effort**		
Π	Π_1	Π_2	**Π**	Π_1	Π_2
Probability	x	$(1 - x)$	**Probability**	y	$(1 - y)$

The principal is risk neutral and maximizes expected profit (net of wages).
The agent is risk neutral and maximizes expected utility, with the (Bernoulli) utility function: $u(w)$, where w is the wage (remuneration) of the agent. In addition to that, the agent dislikes high effort. The disutility of high effort is C, while the disutility of low effort is 0.
The principal does not observe effort, only the profit, so he can only differentiate the wage based on the observed profit. Let's assume that the principal offers a wage of w_1 if the profit turns out to be Π_1 and a wage of w_2 if the profit turns out to be Π_2.

Question: Suppose that the principal wants to induce high level of effort. Write down the principal's problem, i.e. the objective function, the IR constraint for the agent, and the IC constraint (which ensures that the agent indeed chooses high instead of low effort).

Problem 3.
Characterize the set of SPNE in the Ultimatum game:



Problem 4.↑
Consider the Spence-Dixt model discussed in class, with linear demand: $Q = 1 - P$. Assume $w = r = \frac{1}{16}$. Recall that F is the fixed cost that only the entrant has to incur.

a) At what level of F is entry blockaded?
b) At what level of F is entry deterred?
c) At what level of F is entry accommodated?

Problem 5.
Consider the class of trigger strategies for the advertising game:

- Begin the game by playing *N* in stage 1
- Continue to play *N,* if you did not observe any other moves in the past
- If you've observed that at least one of the players played *A* in the previous period, play A for the next *T* periods (*T* is finite and greater than 1), and then return to playing *N*

In other words, begin with cooperation and continue cooperating, but switch to playing A as soon as you observe non-cooperative price by any player, and then go back to cooperation after *T* periods.

Suppose that the two players play identical trigger strategies.
Q: Provide a condition, which assures that this forms an SPNE (check the profitability of one-time deviations during punishment and during cooperation subgames).